Quick Forced Spreading.

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(received 19 July 1993; accepted in final form 19 November 1993)

PACS. 47.15 – Laminar flows (incl. Couette flow).
PACS. 68.15 – Liquid thin films.

Abstract. – We study the thickness of the film entrained on a fibre quickly withdrawn out of a liquid bath. Important deviations from the Landau-Levich law are observed. Two different regimes are successively found and discussed. The first one is explained by considering inertia, and the second one is shown to be a boundary layer effect.

Viscous entrainment of liquid. – The thickness e of the film entrained by a fibre (radius b) out of a bath of a wetting liquid was first derived by Landau, Levich and Deryaguin in the 1940’s [1]. Figure 1 pictures the basis of the theory of forced spreading: as it is pulled out of the reservoir at a velocity V, the fibre strains the static meniscus a length l. This region is referred to as the dynamic meniscus. For a thin fibre (b much smaller than the capillary length), the static meniscus has a zero curvature and the dynamic one (of thickness of order e) has a curvature of order 1/(b + e) − e/l². Hence the pressure balance at the crossover between these two menisci gives l. For thin films (e << b), it reads

\[ l = \sqrt{eb}. \]  

In the (thin) film, the Laplace pressure is \( \Delta p = \gamma/b \), where \( \gamma \) is the surface tension of the liquid. As a consequence, a flow towards the reservoir takes place inside the dynamic meniscus. For small Reynolds numbers, this flow obeys the Poiseuille law:

\[ V = \frac{e^2}{\eta} \frac{1}{l} \frac{\gamma}{b}, \]  

where \( \eta \) is the liquid viscosity. Eliminating \( l \) in eq. (2) by using eq. (1) yields

\[ e = \alpha b Ca^{2/3}, \]  

where \( Ca \) is the capillary number \( (Ca = \eta V/\gamma) \) and \( \alpha \) a constant, explicitly calculated in [1]: \( \alpha = 1.34 \). Equation (3) was experimentally verified for capillary numbers ranging between \( 5 \cdot 10^{-4} \) and 0.1, by coating horizontally nickel wires with viscous silicone oils which always ensured the low-Reynolds-number condition [2].
When the thickness of the film is not negligible compared with the radius of the fibre \((Ca > 0.1)\), \(b\) must be replaced by \((b + \varepsilon)\), which yields instead of (3): \(\varepsilon = b Ca^{2/3}/(1 - b Ca^{2/3})\). This correction to the Landau-Levich equation, first derived by White and Tallmadge [3] and experimentally verified with viscous silicone oils [2], implies that \(\varepsilon\) softly diverges for a capillary number of order 1. At this point, pressures in the film and in the reservoir are equally zero: in the absence of gravity, the reservoir is fully entrained by the fibre.

We focus here on what happens when fibres are coated with water at high velocity \((V\) of order 1 m/s). We show that, in these conditions, the measurements deviate from the Landau-Levich prediction, even though the capillary number remains small compared with 1 \((Ca < 0.05)\). The anomalous behaviours are then classified in two different regimes. Such an experiment reproduces the industrial conditions under which fibres are treated: after extrusion, fibres are pulled at a velocity of 1 to 10 m/s through a dilute aqueous solution. They come out of the bath lubricated, which prevent them from breaking in further operations.

**Visco-inertial entrainment.**

**Experiment.** – The experimental set-up is shown in fig. 2: a horizontal fibre passes through a reservoir-drop trapped inside a teflon tube (inside radius \(R = 2\) mm and length \(L = 34\) mm). This tube is hung from a force captor which continuously weighs the drop. On one side the fibre is wound on a pulley driven by a motor, and on the other side it is maintained under tension by a sinker. As soon as the motor is turned on, the drop begins to diminish. A typical record of the drop weight is displayed in fig. 3. The mass of the drop decreases linearly with time: the thickness of the film, constant in time, is easily deduced from the slope of this line. The set-up is horizontal, so that gravity can be neglected.

A nickel wire \((b = 88.5 \mu m)\) and a molybdenum one \((b = 12.5 \mu m)\) were drawn out of a drop of pure water \((\gamma = 72.8\) dyn/cm and \(\gamma = 1\) cP), at a constant velocity ranging from 20 cm/s to 3 m/s. Using water allows to have quick withdrawal \((V\) of order 1 m/s) with keeping the capillary number much smaller than unity.

The results (thickness of the film divided by the fibre radius as a function of the capillary
Fig. 3. - Experimental record of the mass of the drop as a function of time: here, a nickel wire \((b = 88.5 \mu m)\) drawn out of pure water at \(V = 89 \text{ cm/s}\); from the slope of this line, we get \(e = 13.4 \mu m\). The motor is on between points \(A\) and \(B\). A horizontal division of the grid corresponds to 5 s, and a vertical one to 25 mg.

Fig. 4. - Film thickness (divided by the radius of the wire) as a function of the capillary number for a wire fastly drawn out of a bath of pure water (log-log scale): \(a)\) nickel wire of radius \(b = 88.5 \mu m\) (empty square), \(b)\) molybdenum wire of radius \(b = 12.5 \mu m\) (full square). The dotted line is the Landau-Levich viscous regime (eq. (3)). Experimental data above this line define the visco-inertial regime.

number) are displayed in fig. 4. Above a given capillary number (which depends on \(b\)), it is obvious that the Landau-Levich prediction (eq. (3)) largely underestimates the experimental data. The deviation increases with the capillary number: the thickness of the film diverges, although \(Ca\) remains much smaller than one. The divergence occurs at a larger \(Ca\) for the thinner fibre.

We also checked that a similar divergence regime occurs when using as a liquid a silicone oil of low viscosity \((\gamma = 0.48 \text{ cp})\), tetradecane \((\gamma = 2.18 \text{ cp})\) or water containing surfactants. In the latter case, below the divergence, the film is thicker than predicted by eq. (3): the presence of the surfactants provokes a Marangoni flow, which thickens the film of a factor 2.5 at the maximum [2, 4]. This thickening effect depends strongly on surfactant concentration and slightly on velocity. Thus this effect cannot explain the divergence regime observed here.

**Interpretation.** - Figure 5 allows to understand the origin of this behaviour: the measured thickness (divided by \(e_{LL}\), the Landau-Levich thickness) is plotted as a function of the dimensionless number \(S\) given by

\[
S = \frac{e V^2 b}{\gamma};
\]  

\(S\) is the ratio of the dynamic and capillary pressures (of order \(e V^2\) and \(\gamma/b\), respectively), which hold inside the dynamic meniscus. Figure 5 shows that deviations from Landau-Levich law appear around \(S = 1\), i.e. when inertia has to be taken into account. In particular, it explains why in fig. 4 the divergence takes place at a larger velocity for the molybdenum wire, which is seven times thinner than the nickel one.

We now try to understand the effect of inertia in a simple way. In the dynamic meniscus, the liquid is convected by the fibre at a velocity of order \(V\) (provided that a viscous boundary
Fig. 5. Film thickness (divided by the Landau-Levich thickness derived from (3)) as a function of the dimensionless number $S$, defined by (4), for the data displayed in fig. 4.

Fig. 6. Experimental record of the mass of the drop as a function of time: here a nylon fibre ($b = 100 \mu m$) drawn out of water at $V = 2 m/s$. A the motor is switched on; $B$ the drop is empty. A horizontal division corresponds to 1 s, and a vertical one to 50 mg.

layer could develop). In the same time, the liquid is sucked backwards by capillarity, so that the capillary and the convective terms are of opposite sign in the Navier-Stokes equation which dimensionally writes, instead of eq. (2),

$$\frac{\gamma}{lb} - \frac{\xi V^2}{l} = \frac{\gamma V}{\varepsilon^2}.$$  \hspace{1cm} (5)

Gathering the former equation with eq. (1) (which remains valid) leads to

$$e = \frac{e_{LL}}{(1 - S)^{2/3}}, \hspace{1cm} (6)$$

where $e_{LL}$ is the Landau-Levich thickness. This equation indeed describes the experimental behaviour (fig. 4 or 5). For $S \ll 1$, the thickness of the film increases according to the Landau-Levich model ($e \sim V^{2/3}$). For $S > 0.1$, the thickness increases faster: eq. (6) yields $e \sim V^{8/3}$, comparable with the data (fig. 4). Finally $e$ diverges for a finite velocity ($S$ of order 1).

More detailed calculations can be made, by treating inertia as a perturbation in the classical Landau-Levich theory [5]. These calculations lead to a third-order differential equation for the shape of the dynamic meniscus and then, by matching dynamic with static meniscus, to an implicit expression for $e(V)$. This expression also exhibits a divergence regime at a finite velocity, confirms that this velocity is given by $S$ of order 1, and finally fits the experimental data rather well. Even if such a detailed calculation is necessary, the simple argument given above helps to understand the observed deviations and allows one to predict the scaling laws for $e(V)$ and realistic orders of magnitude for the divergence velocities:
setting $S = 1$ gives a capillary number of 0.01 for the nickel wire ($b = 88.5 \, \mu m$) and of 0.03 for the molybdenum wire ($b = 12.5 \, \mu m$). These values are close to what can be observed in fig. 4. However, the exact value for the divergence depends on the detail of the dynamic meniscus profile, hence on $b$ and $R$, the radius of the tube [5]. Thus $S = 1$ is not exactly the value for the divergence as can be seen in fig. 5.

Comment. – As the fibre is drawn through the reservoir, a viscous boundary layer develops around it. A necessary condition for treating inertia as a perturbation of the viscous entrainment is that the boundary layer must be thick enough. At the exit of the drop, it must be thicker than the dynamic meniscus (of thickness of order $e$). This condition reads

$$\left(\frac{\nu L}{V}\right)^{1/2} \gtrsim e,$$

(7)

with $\nu$ the kinematic viscosity ($\nu = \eta/\rho$) and $L$ the length of the drop. It means that the drop has to be long enough. In our experiments, $e$ is smaller than 110 $\mu m$, $V$ smaller than 1.5 m/s and $\nu$ equal to 0.01 cm$^2$/s. So this condition is always fulfilled, since $L$ is equal to 34 mm, larger than $(e^2 V / \nu)_{\text{max}} = 18 \, \text{mm}$. But the question logically remains: what happens if condition (7) is not satisfied?

The boundary layer regime. – When condition (7) is not satisfied, we propose that the fibre carries with it only the boundary layer, since it is the liquid viscosity which causes the entrainment of liquid by the fibre. According to this proposition, a fibre pulled at still higher velocities should then carry a film of thickness

$$e = \left(\frac{\nu L}{V}\right)^{1/2}.$$

(8)

This equation was verified by making an experiment at fixed velocity ($V = 2 \, \text{m/s}$), while varying the length $L$. The drop was simply allowed to empty, so that $L$ varied from its initial length (51 mm) to zero. The weight of the drop was recorded as a function of time and is displayed in fig. 6.

Fig. 7. – Film thickness (divided by the radius of the fibre) as a function of the capillary number at still larger withdrawal velocities: here a nylon fibre ($b = 110 \, \mu m$) drawn out of water at $V$ ranging from 1 m/s to 5 m/s. Evidence of the boundary layer regime is made for two drop lengths: $L = 34$ mm (plus) and $L = 51$ mm (asterisks). The equation of the lines is $e = 1.1 e_{BL}$, $e_{BL}$ given by (8). The dotted line is the Landau-Levich regime (eq. (3)).
An evident remark is that this plot is no longer a straight line (as it was when condition (7) was satisfied, see for example fig. 2) but rather resembles a parabola. The thickness of the film, which is given by the slope of this curve, decreases with time, i.e. with the shortening of the drop. This fact agrees qualitatively with (8). A more precise verification consists in integrating the flux equation

\[ -\frac{dL}{dt} R^2 = (e^2 + 2eb) V, \]

where \( e \) is given by (8). This equation is particularly easy to solve in the limit \( e \ll b \). \( L \) is found to be proportional to \( t^2 \), by setting \( t = 0 \) for \( L = 0 \).

The region around \( L = 0 \) in fig. 6 was checked to be a parabola. For larger \( L \), \( e \) is not negligible compared with \( b \) and the \( e^2 \)-term must be taken into account in eq. (9). The obtained function \( L(t) \) is shown to fit exactly the experimental curve (fig. 6), providing that \( e = (1.10 \pm 0.05)e_{BL} \), \( e_{BL} \) being the thickness of the boundary layer at the exit of the drop given by (8). The film actually entrained is thus slightly thicker than the boundary layer.

Another experimental verification of the validity of eq. (8) in this high-velocity range (or short-drop range) consisted in doing a series of runs, varying the fibre velocity. From each record similar to fig. 6, the film thickness was deduced for two given drop lengths. The results of this experiment made with a nylon fiber (\( b = 110 \mu m \)) are shown in fig. 7. The two regimes that have been discussed above are clearly visible. First, the thickness diverges due to inertia. Second, when condition (7) is not satisfied, the equation \( e = 1.1e_{BL} \) still matches the decrease of the thickness. Note that this latter regime corresponds to the conditions under which industrial fibres are lubricated.

**Conclusion.** – Forced spreading of water on fibres exhibit three different regimes. For low velocities, the film thickness obeys the classical Landau-Levich equation. Then two anomalous behaviours have been described. The first one is a divergence of the thickness of the film, which occurs when the dimensionless parameter \( S \), characterizing the importance of inertia, is of the order of unity. The second one appears for still higher velocities: the fibre then only entrains the viscous boundary layer, which exists at the exit of the drop.

The crossover between these regimes was not discussed. In fig. 7, we notice a small region between the two regimes where the thickness levels off. This is physically easy to understand: the growth of the dynamic meniscus as \( e \) diverges is stopped at the edge of the tube. Thus the thickness of the film is limited, at a value which depends on \( R \) and \( L \).

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We thank P.-G. DE GENNES for fruitful discussions and D. TEANEY and S. COHEN-ADDAD for their comments.

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